MODIFIED SEMIEMPIRICAL MODEL OF TURBULENCE

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A modified model of the mixing length is constructed. The model adequately accounts for changes in external flow conditions and the history of the flow.

Semiempirical models of turbulence are now fairly widely used in methods of calculating turbulent boundary layers. The most common of these models are shown in Table 1. The models are sufficiently simple in structure and convenient for practical use, but they do not always yield satisfactory results in calculating the characteristics of a turbulent boundary layer — particularly when there are substantial longitudinal pressure gradients.

For a comparative analysis of these turbulence models, we used the experimental data obtained by Schubauer and Spangenbery (experiment 4400) and Bradshaw (experiment 2600) for flows characterized by high pressure gradients with a parameter P* equal to 5-30 and 30-35, respectively (the numerical identification of the experiments corresponds to the materials of the Stanford conference [1]).

Figure 1 compares experimental mixing-length values with values calculated from the formulas in Table 1. The calculated results diverge considerably from each other and from the empirical data, particularly for flow 2600. Thus, substantial errors result from solving turbulent-boundary-layer equations with these models (Figs. 2 and 3), the size of the error depending on the accuracy with which the given model describes the distribution of mixing length over the layer thickness. It should be noted that the experiments of Schubauer and Spangenberg correspond more to the Sebesi and Smith model, while Bradshaw's experiments correspond more to the models of Daisler and McDonald. However, none of these models make it possible to obtain reliable calculated data for both flows (4400 and 2600).

The above analysis shows that the investigated models do not fully reflect the effect of either local external conditions of the flow or its history on the mixing length. It was noted in several studies [2, 6, 7] that turbulence models cannot be based on the

TABLE 1. Models of Mixing Length and Turbulent Viscosity

Serial No.	Model		Author
1	$l = 0{,}08\,\delta\left[1-\left(1-\frac{y}{\delta}\right)^5\right]$		Spaulding [2]
2	$\nu_t = 0{,}41\,yu_1\left[\frac{C_f}{2}\left(1-\frac{y}{\delta}\right)\right]^{0,5}\exp\left(-\frac{y}{0{,}6\,\delta}\right)$		Shablevskii [2]
3	$l = 0{,}1\,\delta\,\frac{1-\exp(-8\,y/\delta)}{1-\exp(-6\,y/\delta)}$		Daisler [3]
4	$l = 0{,}1\,\delta\,\mathrm{th}\left(\frac{Ky}{0{,}1\,\delta}\right),\ K = 0{,}41$		McDonald, Fish [4]
	Inner region	Outer region	
5	$l = 0{,}4\,y\left\{1-\exp\left[\frac{y}{26\,\nu}\times\left(\frac{\tau_W}{\rho}+\frac{y}{\rho}\,\frac{dP}{dx}\right)^{0,5}\right]\right\}$	$\nu_t = 0{,}0168\,u_1\delta^*\times\left[1+5{,}5\left(\frac{y}{\delta}\right)^6\right]^{-1}$	Sebesi, Smith [5]

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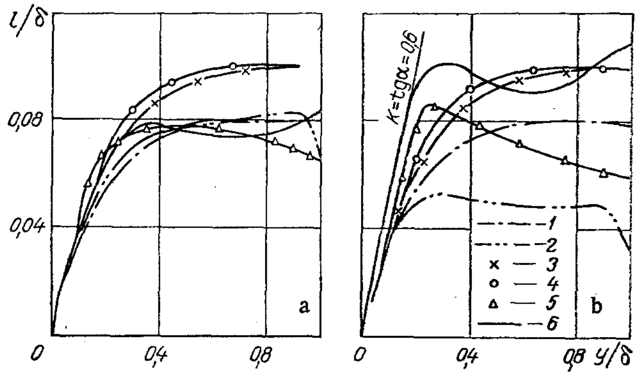

Fig. 1. Mixing length calculated from local values of boundary-layer parameters: a) 4400 (x = 0.51 m); b) 2600 (x = 1.19 m); 1-5) models presented in Table 1; 6) actual distribution.

assumption of a constant value of parameter K = 0.4 (or K = 0.41, as with some authors — see models 2, 4, and 5 in Table 1). According to the data from numerous experiments, the value of K — equal to the slope tangent of the linear section of the graph of mixing-length distribution over the thickness of a boundary layer (Fig. 1) — is significantly affected by a longitudinal pressure gradient: the value of K decreases to 0.35 with a negative pressure gradient and in some cases increases to 0.75 with a positive gradient. The maximum value of the mixing length is also a variable quantity. According to the empirical data, it lies within the range from 0.068δ to 0.115δ and was given a value of 0.08δ or 0.115δ in the investigated models (0.08δ in the Spaulding model and 0.1δ in the Daisler and McDonald models). Thus, the turbulence models presented in the tables are of a special nature, since a limited volume of experimental material which does not embrace different types of gradient flows was used in their construction.

The present work attempts to further improve a mathematical model of mixing length on the basis of analysis of experimental data on a broader class of flows. For analysis and generalizations, we used the experimental data presented in the materials of the Stanford conference considered to be authoritative [1], as well certain data published later [8, 9] and the results of our own experiments.

The following relation was adopted as a basis for construction of our modified model

$$l/\delta = DL\,\text{th}\left(\frac{Ky}{L\delta}\right), \tag{1}$$

where the damping factor

$$D = \left[\text{th}\left(\frac{0.012\,y^+}{K}\right)^2\right]^{0.5}, \tag{2}$$

parameter K = tg α (Fig. 1), $L = l_{max}/\delta$ is the maximum value approached by function (1) at $y/\delta \to 1$; for this quantity, we took the relative mixing length at the point of the first maximum on the experimental curves of l/δ (comparative calculations showed that such a choice for parameter L gives the best results).

The main difference between the modified turbulence model and the previous models is that the parameters K and L, rather than being assumed constant, are functions of the local conditions and history of the flow. The functional relationships for these parameters were determined on the basis of generalization of experimental data from 125 experiments, including 14 different flows described in the materials of the Stanford conference and 11 flows used in [8] and [9] and by us. Also, some of the flows used in the generalization were different in the sign and magnitude of their pressure gradients. The methods of mathematical statistics were used to analyze the sample from the 125 experiments.

The analysis was based on experimental data on mixing length distribution over the thickness of a turbulent boundary layer. In those cases where the Reynolds frictional stresses were not measured directly, it was necessary to determine them using the methods in [2, 10]. The total frictional stresses were found by integrating the boundary-layer equations

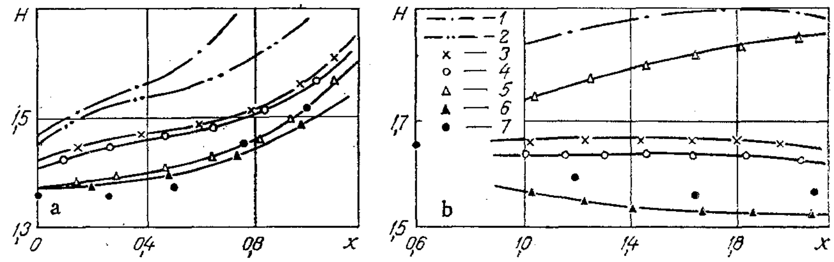

Fig. 2. Form parameter H in the experiments: a) 4400; b) 2600; 1-5) models presented in Table 1; 6) modified model; 7) experiment. x, m.

$$\frac{\tau}{\tau_W} = 1 - \frac{2}{C_f}\,\frac{1}{u_1^2}\left[u_1\,\frac{du_1}{dx}\,y + u\int\limits_0^y \frac{\partial u}{\partial x}\,dy - 2\int\limits_0^y u\,\frac{\partial u}{\partial x}\,dy\right]. \qquad (3)$$

In order to reduce the integration error, the measured velocity distributions over the boundary-layer thickness were approximated by Thompson profiles [11]. This guaranteed reliable calculation of the derivative $\partial u/\partial x$ from two velocity profiles found in the test section and an adjacent section. The derivative $\partial u/\partial y$ was calculated by direct differentiation of the analytical Thompson relations.

The distribution of values of the mixing length over the thickness of the boundary layer $l/\delta = f(y/\delta)$ was plotted for all 125 experiments and the resulting curves were used to find empirical values of K and L. The thus-obtained groups of values of these parameters were then used to establish the generalizing relations.

Starting from the fact that the value of K depends considerably on the pressure gradient and that the change in the gradient is in turn accompanied by a change in the integral characteristics of the boundary layer δ^*, θ and the form parameter H, we first chose the parameters P^*, Re^*, and H as arguments for the multiple regression for K. A preliminary check of the closeness of the correlation between K and the parameters P^*, Re^*, and H showed that the correlation coefficients were equal to 0.815, 0.765, and 0.747, respectively. This shows that the response function is sufficiently dependent on the chosen arguments. Together with this, in constructing the correlation field of the K function, it was found that the empirical values deviated appreciably from the average curve for several equilibrium flows with a positive pressure gradient (flows 2200, 2300, 2500, and 2600, from Clauser's and Bradshaw's experiments, in accordance with the Stanford conference numeration). According to Clauser [12], equilibrium flows are characterized by a constant value of β or H, i.e., $d\beta/dx = 0$ or $dH/dx = 0$. Thus, in addition to the arguments chosen already, we introduced a characteristic of flow equilibrium in the form of the dimensionless parameter $\theta\frac{dH}{dx}$. As a result, the regression equation was sought in the form $K = f\left(H, P^*, \mathrm{Re}^*, \theta\frac{dH}{dx}\right)$.

The form of the correlation was determined by the method of stepwise regression, with a check of the remainder variance. The following relation was obtained as a result:

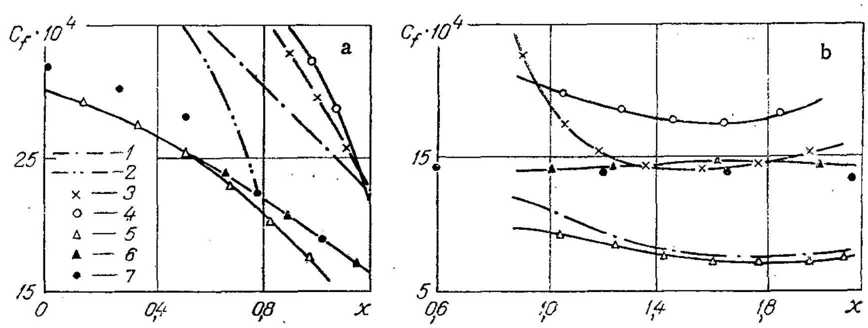

Fig. 3. Friction coefficient in the experiments: a) 4400; b) 2600; 1-5) models in Table 1; 6) modified model; 7) experiment.

$$K = 0{,}386 + \left[10 \ln \frac{\mathrm{Re}^*}{1000} + 10{,}83 \exp\left(-320\,\theta\,\frac{dH}{dx}\right)\right] P^* + 0.00768 \exp\left(-320\,\theta\,\frac{dH}{dx}\right) \ln \frac{\mathrm{Re}^*}{1000}. \quad (4)$$

The rms error of the approximation is 4.6%, the multiple correlation coefficient is equal to 0.97, and its significance according to Student's criterion is 173. This is considerably higher than the critical value for the given sample volume.

The value of L also depends on the pressure gradient. The following expression was obtained for it by the method of stepwise regression

$$L = 0{,}013 + K\left[0{,}0368\,\frac{H}{H-1} + 0{,}0158 \exp\left(-320\,\theta\,\frac{dH}{dx}\right) + 0{,}0269\,K\right] + 0{,}385 \cdot 10^{-7}/\mathrm{Re}^{*2}. \quad (5)$$

Equation (5) approximates the experimental data with a rms error of 7.7%. The multiple correlation coefficient is equal to 0.83 and its significance according to Student's criterion is 28, which exceeds the critical value for the given sample — two.

The accuracy of the turbulence model (1), (2), (4), (5) was checked against an extensive amount of empirical data. The proposed model was used to solve differential equations of a turbulent boundary layer, determine velocity profiles over its thickness in different sections along the flow, and determine integral characteristics of the layer. A comparison was made with the experimental data. The following solution algorithm was used.

By means of the Levi—Liess transform, the motion and continuity equations were reduced to a single differential equation in the current function [5]. The equation was linearized after substitution of finite differences for the derivatives with respect to the longitudinal coordinate. The third derivative of the dimensionless current function was approximated using a five-point model, once having determined the width of the matrix band. The resulting system of linear algebraic equations was solved by the economical (from the point of view of computer operating time) method of rotation [13], which made it possible to calculate the boundary-layer flow on an M-222 computer on grids consisting of up to 170 points over the layer thickness. The accuracy of the calculations with the program was evaluated by solving the equations of a laminar boundary layer for flows along a plate and about a cylinder, which have analytical solutions [14]. The maximum error of the friction coefficients on the wall did not exceed fractions of a percent and the velocity distribution over the thickness of the boundary layer was in nearly complete agreement.

The numerous calculations of turbulent boundary layers completed with the modified model of turbulence showed that good agreement was obtained in all cases with the experimental data with respect to both the velocity distribution over the layer thickness and the integral characteristics. As an example, Figs. 2 and 3 show calculated values of H and C_f (curves 6) for the two flows 2600 and 4400 against which the other models investigated were compared earlier.

The results obtained allow us to recommend the modified model of turbulence to calculate turbulent boundary layers for different gradient flows.

NOTATION

δ^*, displacement thickness; θ, momentum thickness; $H = \delta^*/\theta$, form parameter; C_f, friction coefficient; u_1, velocity on the outer boundary of the boundary layer; δ, thickness of boundary layer corresponding to the distance from the surface in the flow at which the local velocity $u = 0.995u_1$; x, abscissa, reckoned along the surface in the flow; y, ordinate, normal to the surface; l, local value of mixing length, changing over the thickness of the boundary layer; $y^+ = yu^+/\nu$, dimensionless ordinate; $u^+ = \sqrt{\tau_W/\rho}$, dynamic velocity; τ_W, wall shear stress; ρ, density; ν, coefficient of kinematic viscosity; P, pressure; τ, local value of total shear stress at an arbitrary point over the boundary-layer thickness; $\mathrm{Re}^* = u_1\delta^*/\nu$, Reynolds number over the displacement thickness; $P^* = -\delta^* du_1/u_1 dx$, dimensionless parameter accounting for the pressure gradient; $\beta = \delta^* dP/\tau_W dx$, dimensionless equilibrium parameter.

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TURBULENT FLOW IN A BOUNDARY LAYER ON THE INLET AND OUTLET SIDES OF A ROTATING CHANNEL

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Expressions are obtained for approximately determining the shear stresses on the outlet and inlet sides of a rotating channel on the basis of a turbulence energy balance equation, A. N. Kolmogorov's hypothesis, and the Monin—Obukhov similitude theory.

In the flow of a fluid in the rotating channels of turbine rotors, body forces are created by the rotation and curvature of the channel walls. As an example, Fig. 1 shows body forces acting on a particle of fluid on the pressure side of a blade (the outlet side of the channel) in a plane impeller in a radial-flow compressor. The x axis is directed along the blade surface, the y axis is normal to the surface, and the z axis is parallel to the angular velocity vector. It can be seen that in most cases the total body force is negative on the pressure side and positive on the suction side of the blade (the inlet side of the channel).

The different directions of the total body forces on the pressure and suction sides determines the different character of flow in the turbulent boundary layer.

We can use Rayleigh's method to evaluate the stability of the flow and take the Richardson number as the criterion of stability [1, 2]. The total body force acting in the direction of the y axis is equal to (Fig. 1)

$$F = \mp \rho \left(2\,\omega u \pm \frac{u^2}{R} - \omega^2 r \cos\eta\right), \tag{1}$$

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